1 Rolling Dice

(a) Suppose we are rolling a fair 6-sided die. What is the expected number of times we have to roll before we roll a 6? What is the variance?

(b) Suppose we have two independent, fair n-sided dice labeled Die 1 and Die 2. If we roll the two dice until the value on Die 1 is smaller than the value on Die 2, what is the expected number of times that we roll? What is the variance?

2 The Memoryless Property

Let X be a discrete random variable which takes on values in $\mathbb{Z}_+$. Suppose that for all $m,n \in \mathbb{N}$, we have $\mathbb{P}(X > m+n \mid X > n) = \mathbb{P}(X > m)$. Prove that X is a geometric distribution. Hint: In order to prove that X is geometric, it suffices to prove that there exists a $p \in [0,1]$ such that $\mathbb{P}(X > i) = (1-p)^i$ for all $i > 0$.

3 Geometric and Poisson

Let $X \sim \text{Geo}(p)$ and $Y \sim \text{Poisson}(\lambda)$ be independent. random variables. Compute $\mathbb{P}(X > Y)$. Your final answer should not have summations.

4 Fishy Computations

Use the Poisson distribution to answer these questions:

(a) Suppose that on average, a fisherman catches 20 salmon per week. What is the probability that he will catch exactly 7 salmon this week?

(b) Suppose that on average, you go to Fisherman's Wharf twice a year. What is the probability that you will go at most once in 2018?

(c) Suppose that in March, on average, there are 5.7 boats that sail in Laguna Beach per day. What is the probability there will be *at least* 3 boats sailing throughout the *next two days* in Laguna?

5 Combining Distributions

Let $X \sim \text{Poisson}(\lambda), Y \sim \text{Poisson}(\mu)$ be independent random variables. Prove that $X|X+Y$ is binomial. What are the parameters of the binomial distribution? (Hint: Start by expanding $\mathbb{P}(X = k|X+Y = n)$)