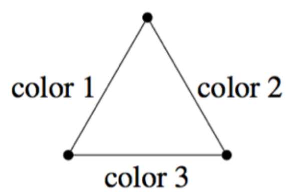
CS 70	Discrete Mathematics and Probability Theory	
Fall 2019	Alistair Sinclair and Yun S. Song	DIS 3

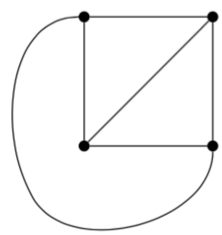
1 Banquet Arrangement

Suppose n people are attending a banquet, and each of them has at least m friends ($2 \leq m \leq n$), where friendship is mutual. Prove that we can put at least $m+1$ of the attendants on the same round table, so that each person sits next to his or her friends on both sides.

2 Edge Colorings

An edge coloring of a graph is an assignment of colors to edges in a graph where any two edges incident to the same vertex have different colors. An example is shown on the left.





(a) Show that the 4 vertex complete graph above can be 3 edge colored. (Use the numbers $1,2,3$ for colors. A figure is shown on the right.)

(b) Prove that any graph with maximum degree $d \geq 1$ can be edge colored with $2d-1$ colors.

(c) Show that a tree can be edge colored with d colors where d is the maximum degree of any vertex.

3 Triangular Faces

Suppose we have a connected planar graph G with v vertices and e edges such that $e = 3v - 6$. Prove that in any planar drawing of G, every face must be a triangle; that is, prove that every face must be incident to exactly three edges of G.

4 True or False

(a) Any pair of vertices in a tree are connected by exactly one path.

(b) Adding an edge between two vertices of a tree creates a new cycle.

(c) Adding an edge in a connected graph creates exactly one new cycle.

(d) We can create a soccer ball by stitching together 10 pentagons and 20 hexagonal pieces, with three pieces meeting at each vertex.