1. [True or False]

(a) ☐ The set of all irrational numbers $\mathbb{R}\backslash\mathbb{Q}$ (i.e. real numbers that are not rational) is uncountable.

(b) ☐ The set of integers x that solve the equation $3x \equiv 2 \pmod{10}$ is countably infinite.

(c) ☐ The set of real solutions for the equation $x + y = 1$ is countable.

For any two functions $f : Y \to Z$ and $g : X \to Y$, let their composition $f \circ g : X \to Z$ be given by $f \circ g = f(g(x))$ for all $x \in X$. Determine if the following statements are true or false.

(d) ☐ f and g are injective (one-to-one) $\Longrightarrow$ $f \circ g$ is injective (one-to-one).

(e) ☐ f is surjective (onto) $\Longrightarrow$ $f \circ g$ is surjective (onto).

2. Consider an $n \times n$ matrix A where the diagonal consists of alternating 1's and 0's starting from 1, i.e. $A[0,0] = 1, A[1,1] = 0, A[2,2] = 1$, etc. Describe an n length vector from $\{0,1\}^n$ that is not equal to any row in the matrix A. (Note that the all ones vector or the all zeros vector of length n could each be rows in the matrix.)

3. Find the precise error in the following proof:
False Claim: The set of rationals r such that $0 \le r \le 1$ is uncountable.
Proof: Suppose towards a contradiction that there is a bijection $f : \mathbb{N} \to \mathbb{Q}[0,1]$, where $\mathbb{Q}[0,1]$ denotes the rationals in $[0,1]$. This allows us to list all the rationals between 0 and 1, with the j-th element of the list being $f(j)$. Suppose we represent each of these rationals by their non-terminating exapansion (for example, $0.4999\ldots$ rather than 0.5). Let d_j denote the j-th digit j-th digit of $f(j)$. We define a new number e, whose j-th digit e_j is equal to $(d_j + 2) \pmod{10}$. We claim that e does not occur in our list of rationals between 0 and 1. This is because e cannot be equal to $f(j)$ for any j, since it differs from $f(j)$ in the j-th digit by more than 1. Contradiction. Therefore the set of rationals between 0 and 1 is uncountable.