1 Mutually Independent Events

There are three mutually independent events A, B and C, with $\mathbb{P}(A) = 2/5, \mathbb{P}(B) = 3/5$ and $\mathbb{P}(C) = 3/10$. Calculate the following.

(a) $\mathbb{P}(A|B)$.

(b) $\mathbb{P}[A \cap B]$.

(c) $\mathbb{P}[A \cup C]$.

(d) $\mathbb{P}[A \cap B \cap C]$.

(e) $\mathbb{P}[A \cup B \cup C]$.

2 Balls and Bins

Throw n balls into n labeled bins one at a time.

(a) What is the probability that the first bin is empty?

(b) What is the probability that the first k bins are empty?

(c) Use the union bound to give an upper bound on the probability that at least k bins are empty.

(d) What is the probability that the second bin is empty given that the first one is empty?

(e) Are the events that "the first bin is empty" and "the first two bins are empty" independent?

(f) Are the events that "the first bin is empty" and "the second bin is empty" independent?

3 Weathermen

Tom is a weatherman in New York. On days when it snows, Tom correctly predicts the snow 70% of the time. When it doesn't snow, he correctly predicts no snow 95% of the time. In New York, it snows on 10% of all days.

(a) If Tom says that it is going to snow, what is the probability it will actually snow?

(b) Let A be the event that, on a given day, Tom predicts the weather correctly. What is $\mathbb{P}(A)$?

(c) Tom's friend Jerry is a weatherman in Alaska. Jerry claims that she is a better weatherman than Tom even though her overall accuracy is lower. After looking at their records, you determine that Jerry is indeed better than Tom at predicting snow on snowy days and sun on sunny day. Give an instance of the situation described above. *Hint: what is the weather like in Alaska?*

4 To Be Fair

Suppose you have a biased coin with $\mathbb{P}(\text{heads}) \neq 0.5$. How could you use this coin to simulate a fair coin? (*Hint*: Think about pairs of tosses.)