CS 70 Discrete Mathematics and Probability Theory
Fall 2019 Alistair Sinclair and Yun S. Song

Quiz 11

1. [True or False] For parts (a) and (b), X is a random variable with $\mathbb{E}[X] = \mu$ and $\text{Var}(X) = \sigma^2$.

(a) ☐ $\mathbb{P}(X \geq 2\mu) \leq (\frac{\sigma}{\mu})^2$.

(b) ☐ Chebyshev's inequality tells us that $\mathbb{P}(|X - \mu| \geq \alpha) \leq \frac{\sigma^2}{\alpha^2}$. From this we can conclude that $\mathbb{P}(X - \mu \geq \alpha) \leq \frac{1}{2} \cdot \frac{\sigma^2}{\alpha^2}$.

(c) ☐ If we toss n balls into $k \geq 3$ bins, the number of balls that land in the first 3 bins is distributed according to a binomial distribution.

2. [Short Answer] Let Z be a random variable which takes values less than 4. We know that $\mathbb{E}[Z] = 2$.

(a) Provide an upper bound on $\mathbb{P}(Z \leq 0)$.

(b) What is a requirement on the variance of Z such that we can guarantee $\mathbb{P}(0 < Z < 4) \geq \frac{1}{2}$?

3. [Long Answer] Let X_i, where $1 \leq i \leq n$, be i.i.d. with mean μ and variance σ^2. Let also $A_n = (X_1 + \cdots + X_n)/n$. Using Chebyshev's inequality, find a 90% confidence interval for μ.