CS 70 Discrete Mathematics and Probability Theory Fall 2019 Alistair Sinclair and Yun S. Song Quiz 12

1. [True/False] For each of the questions, answer TRUE or FALSE. [No need to justify.]

(a) If
$$X \sim \text{Geo}(p)$$
, then $\mathbb{E}[X+m|X>n] = m+n+\mathbb{E}[X]$.

(b) If $X \sim \text{Geo}(p)$, then for $i \ge j$, $\mathbb{P}[X > i | X > j] = (1-p)^{i-j}$

2. [Short Answer] What distribution would best model each of the following scenarios? Choose from Binomial, Poisson, Geometric. No justification needed.

- (a) Number of taxis passing the corner of Euclid Ave and Hearst Ave between 5 pm and 6 pm on a weekday.
- (b) Number of customers who purchase a lottery ticket before someone hits the jackpot.

(c) Number of girls in a family with 6 kids.

3. [Long Answer] CS70 teaching staff are trying to infer how many TAs would need to be present at the final exam to be able to answer questions students might have on the exam. Each TA recorded the statistics of how often students enrolled in their sections ask questions during the last midterm exams. You can assume the students ask questions independently from each other, and the number of questions students ask follows Poisson distribution.

- Edward's students ask N_1 number of questions per every 60 minutes on average.
- Yimings's students ask N_2 number of questions per every 90 minutes on average.
- Jonathan's students ask N_3 number of questions per every 30 minutes on average.

Note that there are no other students outside of these three sections.

- (a) What is the probability that the students ask at least two questions throughout three hour (180 min) final exam?
- (b) If we let Z be the number of questions asked by students from one of the three sections with the least number of questions, what is the probability that $Z \ge 2$?