CS 70 Discrete Mathematics and Probability Theory
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DIS 13

1 Continuous Computations

Let X be a continuous random variable whose PDF is cx^3 (for some constant c) in the range $0 \leq x \leq 1$, and is 0 outside this range.

(a) Find c.

(b) Find the CDF of X.

(c) Find $\mathbb{P}[1/3 \leq X \leq 2/3 \mid X \leq 1/2]$.

(d) Find $\mathbb{E}(X)$.

(e) Find $\text{Var}(X)$.

2 Why Is It Gaussian?

Let X be a normally distributed random variable with mean μ and variance σ^2. Let $Y = aX + b$, where a and b are non-zero real numbers. Show explicitly that Y is normally distributed with mean $a\mu + b$ and variance $a^2\sigma^2$. The PDF for the Gaussian Distribution is $\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. You may want to start by finding the CDF of Y.

3 Lunch Meeting

Alice and Bob agree to try to meet for lunch between 12 PM and 1 PM at their favorite sushi restaurant. Being extremely busy, they are unable to specify their arrival times exactly, and can say only that each of them will arrive (independently) at a time that is uniformly distributed within the hour. In order to avoid wasting precious time, if the other person is not there when they arrive they agree to wait exactly fifteen minutes before leaving. What is the probability that they will actually meet for lunch?