CS 70 Discrete Mathematics and Probability Theory
Fall 2019 Alistair Sinclair and Yun Song

Quiz 1

1. [True or False?] Mark each of the following "True" if it is a valid logical equivalence, or "False" otherwise.

☐ $P \Rightarrow Q \equiv P \vee \neg Q$

☐ $P \Rightarrow Q \equiv (\neg P \Rightarrow \neg Q)$

☐ $P \Rightarrow Q \equiv (Q \wedge P) \vee \neg P$

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2. [True or False?] Let $P(x)$ be a proposition about an integer x, and suppose you want to prove the theorem $\forall x\,(P(x) \Rightarrow Q(x))$. Mark each of the following proof strategies "True" if it would be a valid way to proceed with such a proof, or "False" otherwise.

☐ Find an x such that $Q(x)$ is true or $P(x)$ is false.

☐ Show that, for every x, if $Q(x)$ is false then $P(x)$ is false.

☐ Assume that there exists an x such that $P(x)$ is false and $Q(x)$ is false and derive a contradiction.

☐ Assume that there exists an x such that $P(x)$ is true and $Q(x)$ is false and derive a contradiction.

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3. [Proof] Suppose you have a rectangular array of pebbles, where each pebble is either red or blue. Suppose that for every way of choosing one pebble from each column, there exists a red pebble among the chosen ones. Prove that there must exist an all-red column.