CS 70 Discrete Mathematics and Probability Theory
Fall 2019 Alistair Sinclair and Yun S. Song

Quiz 2

1. [True or False?] For each of the questions below, answer TRUE or FALSE. [No need to justify.]

(a) ☐ Suppose you proved the inductive step for a statement $P(n)$ but then discovered that $P(29)$ is false. Thus, $P(1)$ has to be false.

(b) ☐ Suppose you proved the inductive step for a statement $P(n)$ but then discovered that $P(29)$ is false. Then, we cannot say anything about P(50).

(c) ☐ In a stable marriage instance where there is a man at the bottom of each woman's preference list, the man is paired with his least favorite woman in every stable pairing.

(d) ☐ In a stable marriage instance where there is a man at the top of each woman's preference list, the man is paired with his favorite woman in every stable pairing.

(e) Suppose that, on day k of some execution of a stable marriage algorithm, Alice likes the boy who she currently has on a string more than the boy who Betty has on a string.

☐ It's guaranteed that on every subsequent day, this will continue to be true.

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2. [Inequality.] Prove by induction on n that if n is a natural number and $x > 0$, then $(1+x)^n \geq 1+nx$.

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3. [Stable Marriage.] Suppose that after running the Stable Marriage Algorithm with n men and n women, the pairing that results includes the couple $(1,A)$. Suppose that after a few days man 1 changes his mind, and decides that he does not like woman A as much as he thought he did (i.e. he put her the last person on his preference list). What is the maximum number of rogue couples that result in the existing pairing from such a change to 1's preference list? Give a one or two sentence justification for why the number of rogue couples can be as large as you claim. Also give a one or two sentence justification for why the remaining couples cannot be rogue couples.