

1 True or False

- (a) A graph with k edges and n vertices has a vertex of degree at least $2k/n$.
- (b) If $e \leq 3v - 6$ holds for a graph G , then G is planar.
- (c) An n -dimensional hypercube has an Eulerian cycle if and only if n is even.
- (d) If all vertices of an undirected graph have degree 4, the graph must be the complete graph on five vertices, K_5 .

2 Short Answers

- (a) A connected planar simple graph has 5 more edges than it has vertices. How many faces does it have?
- (b) How many edges need to be removed from a 3-dimensional hypercube to get a tree?

3 Coloring Trees

Prove that all trees with at least 2 vertices are *bipartite*: the vertices can be partitioned into two groups so that every edge goes between the two groups.

[*Hint*: Use induction on the number of vertices.]