CS 70	Discrete Mathematics and Probability Theory	~ -
Fall 2019	Alistair Sinclair and Yun S. Song	Quiz 7

## 1 Diagonalization

How many different ways are there to rearrange the letters of DIAGONALIZATION (15 letters with 3 A's, 3 I's, 2 N's, and 2 O's) without the two N's being adjacent?

## 2 Jelly Bean Factory

A candy factory has an endless supply of red, orange and yellow jelly beans. The factory packages the jelly beans into jars of 100 jelly beans each, with each possible combination of colors in the jar being equally likely. (One possible color combination, for example, is a jar of 56 red, 22 orange, and 22 yellow jelly beans.)

Find *N*, the number of different possible color combinations of jelly beans in a single jar (note that color combinations are undordered).

## 3 Combinatorial Proof I

Provide a combinatorial proof that  $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$ 

## 4 Permutations

- (a) How many permutations of the numbers 1 through *n* are there?
- (b) How many permutations of the numbers 1 through *n* are there such that 1 comes before 2 and after 3? (Assume *n* > 3)
- (c) For each permutation  $\sigma$  of 1 through *n*, let  $\sigma(i)$  denote the value at position *i*. For example, if the permutation is 2,4,1,3, we have  $\sigma(1) = 2$  and  $\sigma(2) = 4$ .

For a fixed  $1 \le k \le n$ , how many permutations  $\sigma$  of 1 through *n* are there where for all i < k,  $\sigma(i) < \sigma(k)$ ? (Express your answer in terms of *n* and *k*.)