CS 70 Discrete Mathematics and Probability Theory
Fall 2019 Alistair Sinclair and Yun S. Song

Quiz 8

1. [True or False]

(a) ☐ For any three events A, B, C, if $\mathbb{P}[A \cap B] > 0$ and $\mathbb{P}[B \cap C] > 0$, then $\mathbb{P}[A \cap C] > 0$.

(b) ☐ For events A, B in a uniform probability space, the probability that neither of the events happen is $1 - \mathbb{P}[A] - \mathbb{P}[B]$.

(c) ☐ For three events A, B, C in a uniform probability space, the probability that *exactly one* of the events happens is $\mathbb{P}[A] + \mathbb{P}[B] + \mathbb{P}[C] - 2\mathbb{P}[A \cap B] - 2\mathbb{P}[A \cap C] - 2\mathbb{P}[B \cap C] + 2\mathbb{P}[A \cap B \cap C]$.

2. [Counting & Probability]

Consider the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70$, where each x_i is a non-negative integer. We choose one of these solutions uniformly at random.

(a) What is the size of the sample space?

(b) What is the probability that both $x_1 \geq 30$ and $x_2 \geq 30$?

(c) What it the probability that either $x_1 \geq 30$ or $x_2 \geq 30$?

3. [Combinatorial Proof]

Give a combinatorial proof that $\binom{n+k-1}{k-1} = \sum_{j=0}^{n} \binom{n-j+k-2}{k-2}$.